STAT 231 — LECTURE 27

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Last Time

We learned about **statistical significance** (drawing conclusions from a $p-$value) vs **practical significance** (drawing conclusions by comparing the point estimate and confidence interval). For determining a test hypothesis, we use the test statistic

$$d = \frac{\overline{Y} - \mu_0}{s/\sqrt{n}}$$

If $p \geq 1 - q$, then μ_0 will be inside of a $100q\%$ confidence interval.

27.1 Hypothesis Test for Unknown sigma

To test $H_0 : \sigma^2 = \sigma_0^2$, we use the test statistic

$$U = \frac{(n-1)S^2}{\sigma^2} \sim \chi^2(n-1)$$

If we assume H_0 to be true, we have

$$\frac{(n-1)S^2}{\sigma_0^2} \sim \chi^2(n-1)$$

For a chi-squared distribution, we will not look at both extremes, we will simply multiply our $p-$value by 2. In other words, for $P(U \leq u) = p$, our $p-$value is $2p$.

- If $P(U \leq u) < 0.5$, our $p-$value is $2P(U \leq u)$
- If $P(U \geq u) < 0.5$, our $p-$value is $2P(U \geq u)$

27.2 Likelihood Ratio Test Statistic

Recall our result from a few lectures ago (lec. 22):

$$\Lambda = -2 \log \left(\frac{L(\theta)}{L(\tilde{\theta})} \right) \sim \chi^2(1)$$

We've seen that this lambda function can be used to connect a confidence interval to a respective likelihood ratio. We can also use this to construct an approximate hypothesis test of $H_0 : \theta = \theta_0$.

If we're estimating our value θ, observe that if H_0 is true, we'd expect to see small values for $\Lambda(\theta_0)$; whereas, large values of $\Lambda(\theta_0)$ provide evidence against H_0. Notice that we're determining how "unusual" a particular outcome is to determine if our null hypothesis is true. Given this test statistic, we can determine a $p-$value for θ! Deriving our formula, we have:

$$\begin{aligned}
p &= P(\Lambda(\theta_0) \geq \lambda(\theta_0); H_0) && && (27.1)\\
&\approx P(U \geq \lambda(\theta_0)) && U \sim \chi^2(1) && (27.2)\\
&= P(Z^2 \geq \lambda(\theta_0)) && Z \sim G(0,1) && (27.3)\\
&= P(Z \leq -\sqrt{\lambda(\theta_0)}) + P(Z \geq \sqrt{\lambda(\theta_0)}) && && (27.4)\\
&= 2 \cdot P(Z \geq \sqrt{\lambda(\theta_0)}) && \text{(By symmetry of } Z) && (27.5)\\
&= 2\left[1 - P\left(Z \leq \sqrt{\lambda(\theta_0)}\right)\right] && && (27.6)
\end{aligned}$$

Example 27.2.1. *Roll Up the Rim to Win from Timmies*

I buy 30 cups from Tim Hortons. 3 of them won. If we let Y = number of winning cups and $H_0 : \theta = \frac{1}{6}$, what conclusions can we make regarding our data? Well, let's determine the $p-$value!

$$\lambda(\theta_0) = -2\log R(\theta_0) = -2\log\left(\frac{\theta_0^y(1-\theta_0)^{n-y}}{\widehat{\theta}^y(1-\widehat{\theta})^{n-y}}\right)$$

Setting our values as $n = 30, y = 3, \theta = \frac{1}{6}, \widehat{\theta} = \frac{3}{30} = 0.1$, we have:

$$-2\log\left(\frac{(1/6)^3(5/6)^{27}}{(0.1)^3(0.9)^{27}}\right) \approx 1.090942$$

We therefore have $\lambda(1/6) = 1.090942$. This means that our $p-$value is:

$$p = 2\left[1 - P\left(Z \leq \sqrt{1.090942}\right)\right] = 0.2962626$$

We conclude that there is **no evidence** based on the observed data that contradicts H_0.