STAT 231 — LECTURE 21

Bartosz Antczak *Instructor: Michael Wallace* *October 30, 2017*

Last Time

We learned about and how to calculate *confidence intervals*. Note that **interval estimation** is determined either by a $100p\%$ likelihood interval or a confidence interval.

Constructing a Confidence Interval

1. Find a value a such that $P(-a \leq Z \leq a) = p$ for $Z \sim G(0, 1)$. Or equivalently $P(Z \leq a) = \frac{1+p}{2}$. Some useful numbers to remember when constructing confidence intervals:
   - 90% CI has $a = 1.645$
   - 95% CI has $a = 1.96$
   - 99% CI has $a = 2.576$
2. A $100p\%$ confidence interval for μ is then
$$\overline{y} \pm a\frac{\sigma}{\sqrt{n}}$$

21.1 Chi-squared Distribution

We will now cover two new distributions: the chi-squared and t distribution. We'll first go over the chi-squared distribution:

Definition

The shape of the **chi-squared distribution** is determined by its so-called *degrees of freedom* denoted as k. We denote the distribution as
$$\chi^2(k)$$

Some properties of the distribution:

- If $W_1, W_2, \ldots, W_n$ are independent random variables with $W_i \sim \chi^2(k)$, then
$$\sum_{i=1}^{n} W_i \sim \chi^2(\Sigma\, k_i)$$
  For example, if $W_1 \sim \chi^2(2), W_2 \sim \chi^2(3)$, then $W_1 + W_2 \sim \chi^2(5)$.
- If $Z \sim G(0, 1)$, then $Z^2 \sim X^2(1)$. This means that if $W \sim \chi^2(1)$, then
$$P(W \leq w) = P\left(-\sqrt{w} \leq Z \leq \sqrt{w}\right) = 2\left[1 - P(Z \leq \sqrt{w})\right]$$
- If $W \sim \chi^2(2)$, then $W \sim \mathit{Exponential}(2)$.