STAT 231 — Lecture 8

Bartosz Antczak *Instructor: Michael Wallace* *September 25, 2017*

Last Time

Consider tossing a coin. The probability that is lands as heads is θ. Note that we *cannot* determine the 'true' value of θ. In order to do that, we'd have to perform an infinite number of trials, which is physically impossible. Instead, we estimate θ through experimentation from a process.

8.1 Maximum Likelihood Estimation

When we estimate for θ, we solve for $\widehat{\theta}$. How do we determine which model to use? We essentially use trial and error:

1. Collect and examine data
2. Propose a distribution model
3. Fit the model to our data
4. Check if the model is a proper fit
5. If required, propose a revised model and return to step 2
6. Draw conclusions from your model

Tip: review the probability models from STAT 230. Know the functions, density functions, etc.

Example 8.1.1. *Travel time*

Define $Y \sim G(\mu, \sigma)$ measures the time it takes for our instructor to commute to UW. Let's estimate the expected commute time $E(Y) = \mu$. We can estimate μ as $\widehat{\mu} = \overline{y}$ (we randomly select n days and measure the mean commute time).
Note: the true value μ is not necessarily equal to the sample mean. Additionally, different draws of the sample data $\{y_1, \ldots, y_n\}$ will result in different values of $\widehat{\mu}$.

Example 8.1.2. *Success or fail*

Consider an experiment with n independent trials with two possible outcomes: $\{S, F\}$ with $P(S) = \theta$. If we have y successes from n' trials, we can estimate θ as

$$\widehat{\theta} = \frac{y}{n'}$$

What methods can we use to estimate θ?

- Perform n trials and measure the number of successes (binomial)
- Keep repeating trials until you observe 5 successes (negative binomial)
- Keep repeating trials until you observe first success (geometric)

Let's stick with the binomial distribution. Suppose we have $n = 25$ trials with 10 successes. Which statement is valid:

a) $P(S) = \theta = 0.4$

b) We don't know $P(S) = \theta$ is equal to, but $\theta = 0.4$ seems a reasonable guess

Response (b) is correct. Recall that we can never be 100% confident about our estimate for θ.
Let Y = number of successes in 25 trials. We define Y Bin$(25, \theta)$:

$$P(Y = y; \theta) = \binom{25}{y} \theta^y (1 - \theta)^{25-y}$$

If we observe 10 successes, we have

$$P(Y = 10; \theta) = \binom{25}{10} \theta^{10} (1 - \theta)^{15}$$

The value of θ that makes the observed data most probable occurs when we maximize on θ (i.e., differentiate P and solve for θ when $P = 0$).