MATH 239 — Tutorial 4

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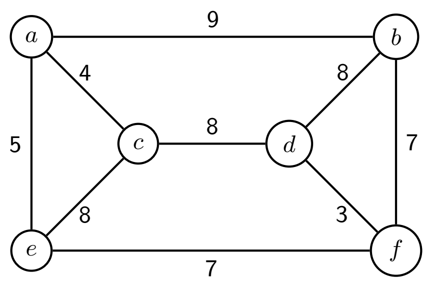
Problem 1 — Building an MST

Given the weighted graph



Construct an MST.

Iteration	Edge added	Vertex Added
0	—	b
1	bf	f
2	df	d
3	ef	e
4	ae	a
5	ac	c

The tree T, where $E(T) = \{bf, df, ef, ae, ac\}$, is an MST, by Prim's algorithm.

Problem 2 — Proving a Statement

Prove that if G is 3-regular and bipartite, it cannot have a bridge.

Proof:

Suppose not. Then G has a bridge $e = uv$, $e \in E(G)$. Let K be the component of $G - e$ that contains u. Since G is bipartite, then K is bipartite. Also, K is nearly 3-regular, except for u, which is 2-regular (since we deleted one of its edges).
Consider the bipartition of K, call them A and B. Since $u \in K$, u is in one of the bipartitions, let's put it in A. Observe that the number of edges in A is the same as in B (by definition of a bipartition). In A, there are $3(|A| - 1) + 2$ edges, and in B, there are $3|B|$ edges. However, the number of edges in A is not divisible by 3 but the number of edges in B are. This means that both partitions have a different number of edges — a contradiction. Therefore, G cannot have a bridge.