# MATH 239 — Lecture 5

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**Note:**

*I was away for this lecture, so today's notes will be substituted with a summary of key points on the respective topics pertaining to this lecture from the provided course notes online. (I'll cover sections 4.7 and 4.8 from the course notes here)*

## 5.1 Equivalence Relations

We define a relation $\mathscr{R}$ between two sets $S$ and $T$ as a subset of $S \times T$. For instance, if $a \in S$ and $b \in T$ (where our main set is $S \times T$), then $a$ and $b$ are *related* or *incident*. These relations can have several properties:

- **Reflexivity**: if each element in $S$ is related to itself
- **Symmetric**: if $a$ is related to $b$, then $b$ is related to $a$ (an example is the relation "divides" for integers. This relation is reflexive (i.e., 7 divides 7) but is not symmetric (i.e., 3 divides 9 does not imply that 9 divides 3))
- **Transitive**: if $a$ is related to$b$ and $b$ is related to $c$, then $a$ is related to $c$

We say a relation is an **equivalence relation** is it satisfies all three of the previous properties.

## 5.2 Connectedness

A graph $G$ is *connected* if, for each two vertices $u$ and $v$, there is a path from $u$ to $v$.

### 5.2.1 Components

A **component** of $G$ is a subgraph $C$ such that

1. $C$ is connected
2. No subgraph of $G$ that properly contains $C$ is connected