MATH 239 — Lecture 22

Bartosz Antczak *March 1, 2017*

Note

I was away for today's lecture, so these notes are my summaries from the course notes, from section 1.4

22.1 Bijections

From last lecture, we mentioned a proposition:

If there exists a bijection from A to B, then $|A| = |B|$

22.1.1 Explanation of our Proposition

So this proposition will be handy when proving that two sets are equal. So how can we show a bijection? Well, we'll do so by providing a function and an inverse function that relates two sets together. If there exists such a function and inverse, then there exists a bijection between the sets and they're equal. We rewrite the previous statements as a theorem:

If function $f : S \to T$ has an inverse, then f is a bijection

Example 22.1.1. *For some $0 \leq k \leq n$, let S be the set of k-subsets of $\{1, \cdots, n\}$, and let T be the set of (n-k)-subsets of $\{1, \cdots, n\}$. Find a bijection between S and T.*

We can do the following mapping:

$$f : S \to T : f(A) = \{1, \cdots, n\} - A \quad \forall A \in S$$

Since $A \in S$, $|A| - S$, and we also see that $f(A)$ $\{1, \cdots, n\} - A$ has $n - k$ cardinality, thus $f(A) \in T$.
In order to "reverse" the operation, we simply need to apply the same operation again, but this time we're applying the operation on the output. So the inverse is $f^{-1} : T \to S$ where for each $B \in T$, $f^{-1}(B) = \{1, \cdots, n\} - B$. We check that for each $A \in S$:

$$f^{-1}(f(A)) = f^{-1}(\{1, \cdots, n\} - A) = \{1, \cdots, n\} - (\{1, \cdots, n\} - A) = A$$

And for each $B \in T$:

$$f(f^{-1}(B)) = f(\{1, \cdots, n\} - B) = \{1, \cdots, n\} - (\{1, \cdots, n\} - B) = B$$

So f^{-1} is indeed the inverse of f which proves that f is a bijection, ergo, both sets are of equal size.