MATH 239 — Lecture 1

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1.1 Course Info

Combinatorics began growing in the fifties and sixties. Some of its progressive development was thanks to some professors here at Waterloo!
This course will cover two major topics (first half before reading week, second half after reading week):

- Graph theory (structure of networks, trees, planarity, matching)
- Enumerative combinatorics (counting, generating series)

We will study these topics, but what will we *learn*?

- new terminology and concepts
- to prove combinatorially (that's what math is: logic and proofs)
- new proof techniques/tricks
- to THINK (combinatorially)

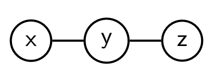
1.2 Graph Theory — Introduction

Let's start with a definition, what is a *graph*? In combinatorics, a **graph** is a set of elements called vertices (or nodes in CS) and a set of pairs of distinct vertices called edges.

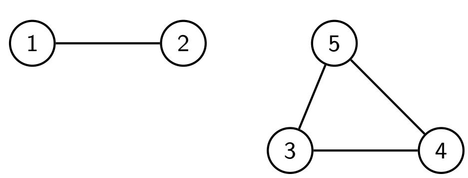
Notation

If G is a graph, we let $V(G)$ denote the set of vertices and $E(G)$ denote the set of edges. For instance, consider graph G with one vertex called a: $V(G) = \{a\}$, $E(G) = \emptyset$
We can also *draw* graphs, where vertices are points and edges are lines/curves connecting the pairs of vertices. We will tend to drawing graphs rather than using the previous notation.

Example 1.2.1. *Let's draw the graph described by $V = \{x, y, z\}$, $E = \{xy, yz\}$*

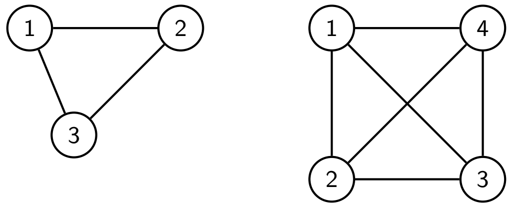


Example 1.2.2. *A disconnected graph (it appears to look like two graphs, but it's actually one). Defined by $V = \{1, 2, 3, 4, 5\}$, $E = \{12, 34, 35, 45\}$. We'll cover this more next week.*

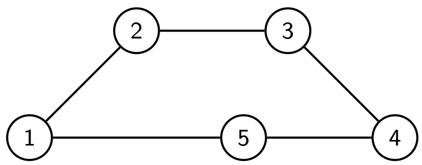


More examples of graphs

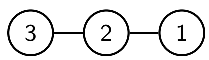
Example 1.2.3. *K_n denotes the complete graph on n vertices, where "complete" means that all pairs of vertices are edges (one vertex is connected to every other vertex). This is our first graph **family**.*
The graphs K_3 and K_4 are shown below:



Example 1.2.4. *C_n denotes the cycle on n vertices. More formally, if $V = \{v_1, v_2, \cdots, v_n\}$, then $E = \{v_1v_2,\, v_2v_3,\, \cdots,\, v_{n-1}v_n,\, v_nv_1\}$. C_5 is shown:*



Example 1.2.5. *P_n denotes the path on n vertices, similar to C_n but there is no cycle. If $V = \{v_1, v_2, \cdots, v_n\}$, then $E = \{v_1v_2,\, v_2v_3, \cdots,\, v_{n-1}v_n\}$. P_3 is shown:*



Example 1.2.6. *$K_{m,n}$ denotes the complete bipartite graph on m and n vertices.*
$V(K_{m,n}) = \{x_1, \cdots, x_m, y_1, \cdots, y_n\}$ with $E = \{x_i\, y_j : \forall\, i, j \mid 1 \le i \le m,\, 1 \le j \le n\}$. $K_{2,3}$ is shown: